MATHEMATICS (Updated January 2013)

A. MEANS OF ASSESSMENT (GRADE 12)

Paper 1	3 hours	[150]
Paper 2	3 hours	[150]
School Based Assessment (SBA)		[100]

400 marks

B. REQUIREMENTS

Included with the papers is a formula sheet.

MATHEMATICS EXAMINATION PAPER 1 (GRADE 12)

Weighting of Content Areas	
Description	**Marks**
Algebra and Equations (and inequalities)	25 ± 3
Patterns and Sequences	25 ± 3
Finance, growth and decay	15 ± 3
Functions and Graphs	35 ± 3
Differential Calculus	35 ± 3
Probability	15 ± 3
TOTAL	**150**

Trigonometric graphs will only be examined in Paper 2.

MATHEMATICS EXAMINATION PAPER 2 (GRADE 12)

Weighting of Content Areas	
Description	**Marks**
Bookwork	6 maximum
Statistics	20 ± 3
Analytical Geometry	40 ± 3
Trigonometry	40 ± 3
Euclidean Geometry and Measurement	50 ± 3
TOTAL	**150**

Questions will be arranged roughly according to level of difficulty from easier to more difficult through the full length of the paper rather than grouped into content focus questions that go from easy to more difficult within a specific content question, i.e. a simple stand-alone trigonometry question could be question 1, but there could be another stand-alone trigonometry question towards the end of the paper that assesses problem solving. This does not exclude a question with some sub-questions that address similar content at a similar level of difficulty.

WEIGHTING ACCORDING TO TAXONOMY OF COGNITIVE LEVEL FOR BOTH PAPER 1 AND PAPER 2

Assessment tasks are designed to the following weighting

Level		**%**
1	Knowledge	**20 (± 3)**
2	Routine procedures	**30 (± 3)**
3	Complex procedures	**35 (± 3)**
4	Problem solving and investigations - reasoning and reflecting	**15 (± 3)**
	Total	**100**

SCHOOL BASED ASSESSMENT (SBA)

SBA comprises 25% of the total assessment for the National Senior Certificate. The requirements for the school-based component of the senior certificate assessment are outlined in the table below.

LEARNER FILE REQUIREMENTS FOR GRADE 12

Descriptions	**Weighting**	**Mark**
2 short items chosen from the selection	2 × 10	20
1 long item chosen from the selection	30	30
Two tests: Standardised and at least 45 minutes to an hour in duration in controlled environment, consisting of Paper 1 and Paper 2.	2 × 10	20
Grade 12 Preliminary Examination consisting of Paper 1 and Paper 2	2 × 15	30
Total Marks:		100

Work in the learner file must be done in the current academic year. Tasks tackled in grade 10 and grade 11 may not be submitted in the grade 12 learner file. All schools must make available the Grade 12 SBA evidence should it be required by the IEB or Umalusi.
These subject assessment guidelines must be read in conjunction with the IEB manual for the moderation of School Based Assessment (2011) available at www.ieb.co.za.

Grade 10 and 11

Although SBA in grade 10 and 11 will not be monitored by the IEB, it is proposed that SBA in grade 10 and 11 follow the same format as that for grade 12.

Short Items
Learner must select two from this list

- Translation Task
- Question setting
- Formula Sheet
- Teaching a lesson
- Written explanations
- Guided Discovery
- Skills analysis
- Olympiads
- Investigation
- Journal
- Performance of a Song, dance or speech*
- Meta Cog
- Lesson to a friend
- Error spotting
- Computer products*
- Problem solving of an untraditional nature
- Cheat Sheet
- Modelling*

These items require at least 45 minutes to complete. Two different tasks in this category must be selected and weighted 10 marks each. These tasks should in most cases be done under controlled circumstances. The exceptions to this rule would be those marked with an asterix*. See above. Those marked with an asterix would need to be monitored so that the teacher can establish that this is indeed the work of the learner. Evidence of monitoring should be provided (e.g. evidence of teacher / learner meetings and formative assessment or orals where the teacher talks to the learner to establish that the learner has ownership of the work.)

Long Items
These require about 5 hours to complete and in some cases some contact time in class.

• Projects: Multi-faceted
• Discovery: a sizeable piece of work
• Computer products i.e. using Autograph to investigate elements of calculus
• Investigation: open ended and requiring significant effort
• Modelling a real life situation
• Revision Booklet on a significant piece of work (e.g. Functions)
• Formula Sheet

Standardised tests in a controlled environment.

These tests are to be written under controlled conditions within a specified period of time. One test should cover content/skills Paper 1 and the second test should cover content/skills Paper 2.

Examinations

Grade 10	Mid-year examination. (one paper is acceptable)
Grade 11	Mid-year examination. (two papers are suggested)
Grade 12	One examination (Paper 1 and Paper 2) according to the end of year format.

More detailed descriptions of the School Based Assessment tasks listed under short and long items above

1. Translation Task
   A translation task tests the ability of a learner to change mathematical notation into language and his/her ability to change language back to mathematical notation. It could also include graphs to equations and equations to graphs. This is good practice and should happen in all sections of work. If this is to be used as a short piece it should include both aspects of the translation.

2. Journal
   A journal requires that the learner writes and reflects on the learner's own practice. It is a metacognitive activity.

Here are three possible categories.

◊ The learner identifies questions which caused his/her problems and then writes notes discussing the problem and the solution.

◊ A teacher driven journal item instructs learners to reflect on a given situation or problem and to use language to do this e.g. the teacher gives an alternate or erroneous solution and asks learners to

- ♦ comment on the alternate solution (e.g. plusses and minuses of using) or
- ♦ identify and explain an erroneous solution and give a correct solution

◊ The learner is asked to reflect and comment on real world situations requiring a mathematical interpretation; this could be an article in a magazine or newspaper.

3. Question Setting
This could be used as a short item and can take the form of one question and even be done in pairs**.** It could be a long item and have several facets to the task**.** Learners could set a question or test and provide a memo. The learner then writes another learner's test and marks the test of the person who wrote his/her test.

4. Performance (song, dance, speech and poster)
This is a very open ended topic which often encompasses innovative ideas. The poster is perhaps most open to abuse. It could also allow for creative work requiring other skills, namely, a cross number. Then we have the more off the wall type presentation where songs, skits and dance are used, e.g. a rap song containing the essence of a section of Mathematics. A poster can also be used to report an investigation. Such a poster can, for example, describe the problem, the method followed, the results found and give the conclusion.

5. Formula Sheets
This can be used as a short or long item. As a short item the learners could be asked to redesign the formula sheet justifying the changes they make. They could also be given a selection of formulae and asked to comment on how they are used. For a longer item the whole sheet could be used and they could be asked to provide examples on how these formulae are used, as well as redesigning the sheet.

6. Teaching a lesson
A nice way of handling this is to make it a group exercise or to let learners work in pairs. They could be given parts of a section of work not yet discussed in class to prepare and present. The teacher and their peers could then assess them against a set of freely available criteria designed to assess the achievement of the topics.

7. A lesson to a friend
Typically the learner writes to a friend explaining of a portion of work**.** This would be based on work handled in class and would give the learner the opportunity to reflect and clarify their own thinking. It would also require them to learn to express mathematical ideas using written language.

8. Metacog
A metacog/ mind map would normally constitute a short item. When an extra part is added to the task it could become a long item. Metacogs should be done under controlled circumstances so that we really are getting to the learner's knowledge about the sections. The learner's deeper thinking could be tested with this metacog. E.g. Use a metacog / Mind map to show your understanding of the function $f(x) = 2ax^2 - 5x$

9. Error spotting
This can also be either a short or a long item. The difference can largely be established by the cognitive demands of the task and the time required. This has also been mentioned under journals.

10. Computer Products
These can take many forms.

◊ They can be guided discovery. An ideal method for this task would be to use Autograph as this would take care of all the graphs without being too time consuming.

◊ Independent work and research with the use of geometer sketchpad.

11. Skills Analysis
This is a difficult one to describe as it could be described in a multitude of ways.

◊ A possible interpretation would involve challenging the learners to 'stretch' their understanding and challenge their perceptions, e.g. describe how to solve x/a > 1.

◊ It could also involve the learner making sense of some new mathematics and then using this knowledge to do some problems.

12. Problem solving of an non-traditional nature
This can take on many forms and to try and describe it would only limit the possibilities. One such example would be: Discuss as many ways as possible how the solution of $\frac{1}{x+2} > -x$ can be found. The answer to this question could involve algebra or graphs or a spreadsheet or trial and error.

13. Olympiads
Enter competitions like the Harmony Mathematics Olympiad or U.C.T. competition or even the Pretoria University Mathematics Competition. Submit the scripts and question paper as an SBA item.

14. Cheat Sheet
This is only suitable as a short piece. The learners must analyse and synthesise a section extracting the essential aspects they are required to know. They must then represent this information on an A5 sheet of paper. This activity is similar to a mind map or metacog.

Number 15, 16, 17 and 18 should all include some of the ideas outlined below.
These should include some of the following steps:

◊ identify a problem to be solved

◊ conjecture after some preliminary work (that is generated from the investigative question)

◊ collect data / information

◊ select and arrange (manipulate or display) relevant data

◊ draw conclusions

◊ finalise theory

◊ write the report indicating all the steps and the process undertaken to come to the conclusion.

◊ a maximum of three weeks should be allocated.

◊ learners can be assessed individually and collectively.

15. Investigation
    These can be used as short or long pieces. The activity should involve an investigation of a pattern or trend, i.e. something that can be grown into a bigger piece of Mathematics. It should involve testing, conjecture and finally a conclusion. Sections like Number Theory and Patterning provide us with a host of good material.

16. Discovery (Guided Investigation)
    This is also a kind of investigation but we now want them to arrive at some conclusions we have already identified. They certainly can take it further but we have a specific reason for asking them to investigate the topic.

17. Projects (multifaceted)
    This would involve at least three activities ranging over the four difficulty levels. The learner would be required to produce a significant piece of work.

18. Modelling
    This would involve finding the mathematics in a real life situation.
    An example of this would be investigating mathematically a newspaper report that people in a neighbouring country have to take a wheelbarrow of money to pay for their bread. This would involve the learner working out the volume the wheelbarrow could hold and then working out the volume of a wad of notes. In this way they would establish the feasibility of the claim while using mathematical models to make sense of the situation. Another modelling problem could involve an examination of rugby scores and how they would be influenced if the value of the points for a conversion were changed.

19. Revision Booklet
    This would involve a theoretical section where theory was presented and examples explained and then a section where graded examples were given. The learners would be expected to provide full solutions to their exercises. The teacher needs to see evidence of that the learners engaged in metacognitive processes during the completion of the task, in their choice of examples and explanation.

C. INTERPRETATION OF REQUIREMENTS

MATHEMATICS
INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1}\ ;\ r \neq 1 \qquad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$A = P(1 + ni) \qquad A = P(1 - ni)$$

$$A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$F = x\left[\frac{(1+i)^n - 1}{i}\right] \qquad P = x\left[\frac{1 - (1+i)^{-n}}{i}\right]$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}\ ;\ \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\overline{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum(x - \overline{x})(y - \overline{y})}{\sum(x - \overline{x})^2}$$

D. ADMINISTRATIVE AND SUPPORT MATERIAL

1. Appendix A: Summary of candidates' assessment
2. Appendix B: Summary of assessment
3. Appendix C: Internal school based assessment checklist
4. Appendix D: SBA moderation form
5. Appendix E: Letter from the principal
6. Appendix F: Curriculum content and clarification

APPENDIX A: SUMMARY OF CANDIDATES' ASSESSMENT

INDEPENDENT EXAMINATIONS BOARD

NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS SBA

		Short Items (45 mins +)			**Long item (5 hrs.)**		**Tests (45 mins +)**			**Prelims**			**FINAL**
N.B. Please indicate what total each assessment was marked out of. Include actual marks under the relevant columns. Under the sub-total column give the total for that section as a weighted percentage. You may write down rounded subtotals but the final calculation must be calculated without rounding.		Short 1 Out of:	Short 2 Out of:	**Sub-Total**	Long item Out of:	**Sub-Total**	Test 1 Out of	Test 2 Out of	**Sub-Total**	Paper 1 Out of	Paper 2 Out of	**Sub-Total**	
										150	150		
EXAM No.	**Name**	10%	10%	**20%**	30%	**30%**	10%	10%	**20%**	15%	15%	**30%**	**100%**

DECLARATION BY THE CANDIDATE'S TEACHER: I ____ (print name and title of teacher) at ____________________________(print name of school) declare that the work provided by these candidates has been monitored and checked for plagiarism.

Signed (Teacher): ___________________________ Date: ___________________________

APPENDIX B: SUMMARY OF ASSESSMENT

INDEPENDENT EXAMINATIONS BOARD

NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS SBA

NSC Summary of Assessment: Mathematics

To be filled in by the candidate, controlled by the teacher
and included as the first page of the learner's file.

Name of candidate: ____

Candidate's Examination Number

Short Items (recommended 45 min)		**Mark**	**Out of**	**Mark as %**
1				
2				
Long Item (recommended 5 hours)		**Mark**	**Out of**	**Mark as %**
1				
Standardised Tests (recommended 45 to 60 min)		**Mark**	**Out of**	**Mark as %**
1				
2				
Preliminary Examinations		**Mark**	**Out of**	**Mark as %**
1	**Paper 1**			
2	**Paper 2**			

		Candidate's Marks as %		**Max**	**Final**
Alternate Assessment	**Short**			**20**	
	Long			**30**	
Tests	**Formal**			**20**	
Examinations	**Paper 1**			**15**	
	Paper 2			**15**	
FINAL CASS				**100**	

DECLARATION BY THE CANDIDATE:

I, ______________________________________print full names) declare that all external sources used in my file have been properly referenced and that the remaining work contained in this file is my own original work. I understand that if this is found to be untrue, I am liable for disqualification from the Senior Certificate Examination.

Signed: Date:

APPENDIX C: INTERNAL SCHOOL BASED ASSESSMENT CHECKLIST

INDEPENDENT EXAMINATIONS BOARD

NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS SBA

Teacher Name		**Teacher Name**	
Teacher Name		**Teacher Name**	

	Criteria to check	
1.	Files as flat as possible; no plastic sleeves; file dividers marked for each section in order of the summary sheet.	
2.	Have read the moderator's previous years report and have monitored that the recommendations are carefully considered and implemented by the teachers concerned.	
3.	Where more than one teacher is teaching a grade, the examination papers are moderated by another using an appropriate set of criteria (e.g. as in the examination requirements)	
4.	Has made sure that internal moderation has occurred and that moderation of standards across learner tasks in the classes of different teachers has been done. (Same question papers across classes, same markers, similar tasks moderated to ensure they make similar demands on learners, and so on.)	
5	Have monitored the relationship between SBA averages and examination or test averages. Try to aim for the final SBA mark average for the group being between 5 and 10% above the final examination average.	
6.	Have checked that all items required for SBA are included in the file.	
7.	Have checked that the full set of marks, and breakdown, is included in the teacher file.	
8.	Have checked that at least three types of alternate assessment have been done in each learners files.	

________________________ ____________________

HOD: Mathematics Date

APPENDIX D: SBA MODERATION FORM

INDEPENDENT EXAMINATIONS BOARD

NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS SBA

This form is to be used for Regional and National Moderation:

CENTRE:	CENTRE NUMBER:

NUMBER OF TEACHERS IN THE MATHEMATICS DEPARTMENT		
NAMES OF TEACHERS		

TEACHER FILE				
TYPE OF MODERATION:	CLUSTER	REGIONAL	NATIONAL	*Circle correct option*
ADMINISTRATION				**COMMENT**
IEB RANDOM SELECTION LIST	LIST PRESENT	YES	NO	
	SIGNED BY PRINCIPAL	YES	NO	
	REQUESTED FILES PRESENT	YES	NO	
IEB MARK LIST	MARK LIST PRESENT	YES	NO	
	SIGNED BY HOD	YES	NO	
	SIGNED BY PRINCIPAL	YES	NO	
	MARKS ENTERED CORRECTLY	YES	NO	
	MARKS CORRESPOND WITH PUPIL FILES	YES	NO	
COMPOSITE MARK LIST	PRESENT	YES	NO	
	CORRECT MARK CALCULATION	YES	NO	
LETTER FROM PRINCIPAL		YES	NO	

NB:

1. It is expected that tasks are **internally moderated** and evidence of this process should be given in the teacher file. Where there is only one teacher in the department they will be expected to buddy with a teacher in another school in the same situation.
2. It is expected that the analysis grids be provided – it is not expected that every task conforms exactly to the suggested distribution but it is sufficient to show an awareness of a need to distribute cognitive demand over all four levels.

SHORT ITEM OPTIONS – NO ITEM MAY BE SELECTED MORE THAN ONCE ALL LEARNING LEVELS (FROM 1 TO 4) MUST BE COVERED FOR EACH SHORT ITEM		
Translation	Formula Sheet	Olympiad with full solutions
Guided discovery / Investigations	Error Spotting	Metacog / Cheat Sheet
Other	Other	Other

SHORT ITEMS						
SHORT ITEM 1 10%	LEVELS COVERED	1	2	3	4	Comment on analysis grids, levels and suitability of the piece.
	ACCEPTABLE LENGTH	YES		NO		
	MEMORANDUM	YES		NO		
SHORT ITEM 2 10%	LEVELS COVERED	1	2	3	4	Comment on analysis grids, levels and suitability of the piece.
	ACCEPTABLE LENGTH	YES		NO		
	MEMORANDUM	YES		NO		

LONG ITEM OPTIONS – THE COMPLETION OF THESE TASKS SHOULD TAKE A MINIMUM OF 5 HOURS (APPROXIMATELY 300 MARKS)		
Guided discovery / Investigations	Modelling a real life situation	Project Multifaceted
Other	Other	Other

LONG ITEM						
LONG ITEM 30%	LEVELS COVERED	1	2	3	4	Comment on analysis grids, levels and suitability of the piece.
	ACCEPTABLE LENGTH	YES		NO		
	MEMORANDUM	YES		NO		
TESTS						
TEST 1 10%	LEVELS COVERED	1	2	3	4	Comment on analysis grids, levels and suitability of the test.
	ACCEPTABLE LENGTH	YES		NO		
	MEMORANDUM	YES		NO		
TEST 2 10%	LEVELS COVERED	1	2	3	4	Comment on analysis grids, levels and suitability of the test.
	ACCEPTABLE LENGTH	YES		NO		
	MEMORANDUM	YES		NO		

PRELIMINARY EXAMINATION						
PAPER 1 15%	LEVELS COVERED	1	2	3	4	Comment on analysis grids, levels and suitability of the examination.
	TOPICS	1	2			
	ACCEPTABLE LENGTH	YES		NO		
	MEMORANDUM	YES		NO		
PAPER 2 15%	LEVELS COVERED	1	2	3	4	Comment on analysis grids, levels and suitability of the examination.
	TOPICS			3	4	
	ACCEPTABLE LENGTH	YES		NO		
	MEMORANDUM	YES		NO		

COMMENT:

LEARNER FILE			COMMENT
CENTRE NUMBER DISPLAYED	YES	NO	
CORRECT FILE; NO PLASTIC SLEEVES; FILE DIVIDERS	YES	NO	
ADMINISTRATION			
COPY OF LEARNER FILE MARK SHEET	YES	NO	
DECLARATION OF AUTHENTICITY	YES	NO	
TWO SHORT ITEMS	YES	NO	
ONE LONG ITEM	YES	NO	
TEST 1 COVERING CONTENT FROM PAPER 1	YES	NO	
TEST 2 COVERING CONTENT FROM PAPER 2	YES	NO	
PRELIMINARY EXAMS PAPER 1 AND 2	YES	NO	

GENERAL COMMENT

MODERATOR: ______________________

DATE: ______________________

APPENDIX E: LETTER FROM THE PRINCIPAL

INDEPENDENT EXAMINATIONS BOARD
NATIONAL SENIOR CERTIFICATE EXAMINATION
MATHEMATICS SBA

SCHOOL: ________________________
ADDRESS: ________________________
____
____

The IEB
PO Box 875
Highlands North
2037

Dear IEB Moderator

RE: SCHOOL BASED ASSESSMENT AND MODERATION OF SBA IN GRADE 12

MATHEMATICS

We certify that

Teachers of the same subject have ensured that	Circle your response	
they have met regularly to reflect on and discuss issues of standardisation	YES	NO
the assessment tasks they have set learners are of the required standard	YES	NO
the memoranda they have used for marking are accurate and functional	YES	NO
the tasks learners have completed meet the criteria described in the IEB Subject Assessment Guidelines	YES	NO
marking is complete and of the appropriate standard	YES	NO
all administrative procedures have been correctly completed	YES	NO
all information on the 1st page of the file (Appendix B) in each learner's file is complete and correct	YES	NO

________________________ TEACHER

DATE: __________________

________________________ PRINCIPAL

DATE: __________________

APPENDIX F: CURRICULUM CONTENT AND CLARIFICATION

MATHEMATICS

- **N.B. Sequencing and pacing is only a guideline**.
- Examples of cognitive demand in questions may be accessed at <www.ieb.co.za> under the National Senior Certificate – Analysis Grid.

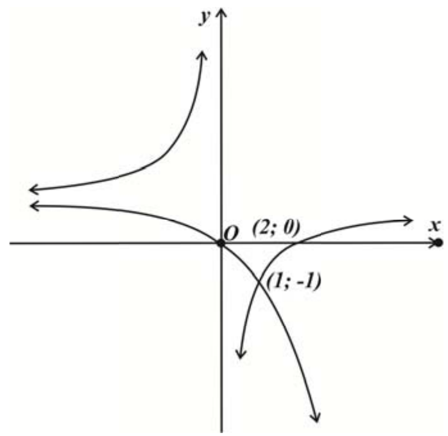
GRADE 10: TERM 1			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
3	**Algebraic expressions**	1. Understand that real numbers can be rational or irrational. 2. Establish between which two integers a given simple surd lies. 3. Round real numbers to an appropriate degree of accuracy. 4. Multiplication of a binomial by a trinomial. 5. Factorisation to include types taught in grade 9 and: • trinomials • grouping in pairs • sum and difference of two cubes 6. Simplification of algebraic fractions using factorisation with denominators of cubes (limited to sum and difference of cubes).	**Examples:** 1. Factorise fully: 1.1 $m^2 - 2m + 1$ (revision) — Learners must be able to recognise the simplest perfect squares. 1.2 $2x^2 - x - 3$ — This type is routine and appears in all texts. 1.3 $\frac{y^2}{2} - \frac{13y}{2} + 18$ — Learners are required to work with fractions and identify when an expression has been 'fully factorised'. 2. Simplify: $\frac{1-2x}{4x^2-1} - \frac{x+4}{2x^2-3x+1} + \frac{1}{1-x}$

[Adapted from: Curriculum and Assessment Policy Statement (CAPS), Mathematics Grade 10 - 12, Department: Basic Education © 2011]

2	**Exponents**	1. Revise laws of exponents learnt in Grade 9 where $x, y > 0$ and $m, n \in Z$: • $x^m \times x^n = x^{m+n}$ • $x^m \div x^n = x^{m-n}$ • $(x^m)^n = x^{mn}$ • $x^m \times y^m = (xy)^m$ Also by definition: • $x^{-n} = \frac{1}{x^n}$; $x \neq 0$, and • $x^0 = 1$, $x \neq 0$ 2. Use the laws of exponents to simplify expressions and solve equations, accepting that the rules also hold for $m, n \in Q$.	**Examples:** 1. Simplify: $(3 \times 5^2)^3 - 75$ 2. Simplify: $\frac{9^x - 1}{3^x + 1}$ 3. Solve for x: 3.1 $2^x = 0{,}125$ 3.2 $2x^{\frac{3}{2}} = 54$ 3.3 $3^{x+1} + 3^{x-1} = \frac{10}{9}$ 3.4 $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 18 = 0$
1	**Numbers and patterns**	Patterns: Investigate number patterns leading to those where there is a constant difference between consecutive terms, and the general term is therefore linear.	**Examples:** 1. Determine the 5th and the nth terms of the number pattern 10; 7; 4; 1; … (There is an algorithmic approach to answering such questions, $T_n = a + (n-1)d$ is not used in Grade 10.) 2. If the pattern MATHSMATHSMATHS … is continued in this way, what will the 267th letter be? It is not immediately obvious how one should proceed, unless similar questions have been tackled.
2	**Equations and Inequalities**	1. Revise the solution of linear equations. 2. Solve quadratic equations (by factorisation). 3. Solve simultaneous linear equations in two unknowns. 4. Solve word problems involving linear, quadratic or simultaneous linear equations.	**Examples:** 1. Solve for x: $\frac{2x-3}{3} - 3x = \frac{2x}{6}$ 2. Solve for m: $2m^2 - m = 1$ 3. Solve for x and y: $x + 2y = 1$; $\frac{x}{3} + \frac{y}{2} = 1$

		5. Solve literal equations (changing the subject of a formula). 6. Solve linear inequalities and show solution graphically. Use of Interval Notation is required.	4. Solve for r in terms of V, π and h: $V = \pi r^2 h$ 5. Solve for x: $-1 \leq 2 - 3x < 8$
3	**Trigonometry**	1. Define the trigonometric ratios $\sin\theta$, $\cos\theta$ and $\tan\theta$, using right-angled triangles. 2. Extend the definitions of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for $0° \leq \theta \leq 360°$. 3. Define the reciprocals of the trigonometric ratios, $\text{cosec}\theta$, $\sec\theta$ and $\cot\theta$, using right-angled triangles (these three reciprocals should be examined in grade 10 only) 4. Derive values of the trigonometric ratios for the special cases (without using a calculator) $\theta \in \{0°;30°;45°;60°;90°\}$. 5. Solve two-dimensional problems involving right-angled triangles. 6. Solve simple trigonometric equations for angles between $0°$ and $90°$. 7. Use diagrams to determine the numerical values of ratios for angles from $0°$ to $360°$.	**Comment:** It is important to stress that: similarity of triangles is fundamental to the trigonometric ratios $\sin\theta$, $\cos\theta$ and $\tan\theta$. **Examples:** 1.If $5\sin\theta + 4 = 0$ and $0° < \theta < 270°$, calculate the value of $\sin^2\theta + \cos^2\theta$ without using a calculator. 2. Trigonometric ratios are independent of the lengths of the sides of a similar right-angled triangle and depend (uniquely) only on the angles, hence we consider them as functions of the angles; and 3. doubling a ratio has a different effect from doubling an angle, for example, generally $2\sin\theta \neq \sin 2\theta$ **Example:** 1. Let $ABCD$ be a rectangle, with $AB = 2$ cm. Let E be on AD such that $A\hat{B}E = 45°$ and $B\hat{E}C = 75°$. Determine the area of the rectangle. 2. Determine the length of the hypotenuse of a right-angled triangle, ABC, where $\hat{B} = 90°$, $\hat{A} = 30°$ and $AB = 10$ cm. **Comment:** Solve equation of the form $\sin x = c$, or $a\cos x = c$, or $\tan ax = c$, where a and c are constants. **Example:** Solve for x: $4\sin x - 1 = 3$ for $x \in [0°;90°]$

GRADE 10: TERM 2

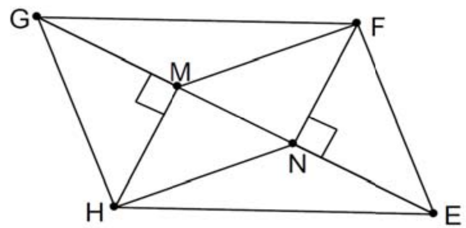
No. of weeks	Topic	Curriculum statement	Clarification
4	**Functions**	1. The concept of a function, where a certain quantity (output value) uniquely depends on another quantity (input value). Work with relationships between variables using tables, graphs, words and formulae. Convert flexibly between these representations. **Note:** that the graph defined by $y = x$ should be known from Grade 9. 2. Point by point plotting of basic graphs defined by $y = x^2$, $y = \frac{1}{x}$ and $y = b^x$; $b > 0$ and $b \neq 1$ to discover shape, domain (input values), range (output values), asymptotes, axes of symmetry, turning points and intercepts on the axes (where applicable). 3. Investigate the effect of a and q on the graphs defined by $y = a.f(x) + q$, where $f(x) = x$, $f(x) = x^2$, $f(x) = \frac{1}{x}$ and $f(x) = b^x$, $b > 0$, $b \neq 1$.	**Comments**: 1. A more formal definition of a function follows in Grade 12. At this level it is enough to investigate the way (unique) output values depend on how input values vary. The terms independent (input) and dependent (output) variables might be useful. 2. After summaries have been compiled about basic features of prescribed graphs and the effects of parameters a and q have been investigated: a: a vertical stretch (and/or a reflection about the x axis) and q a vertical shift. The following examples might be appropriate: **Examples:** 1.Sketched below are graphs of $f(x) = \frac{a}{x} + q$ and $g(x) = nb^x + t$. The horizontal asymptote of both graphs is the line $y = 1$. Determine the values of a, b, n, q and t.  **Remember:** that graphs in some practical applications may be either discrete or continuous.

		4. Point by point plotting of basic graphs defined by $y=\sin\theta$, $y=\cos\theta$ and $y=\tan\theta$ for $\theta\in[0°;360°]$. 5. Study the effect of a and q on the graphs defined by: $y=a\sin\theta+q$; $y=a\cos\theta+q$; and $y=a\tan\theta+q$ where $a,\ q\in Q$ for $\theta\in[0°;360°]$. 6. Sketch graphs, find the equations of given graphs and interpret graphs. **Note**: Sketching of the graphs must be based on the observation of number 3 and 5.	**Example:** Sketch the graph defined by $y=-\sin x+\frac{1}{2}$ for $x\in[0°;360°]$. **Note:** Trig. graphs will be examined in paper 2 only.
4	**Euclidean Geometry**	1. Revise basic results established in earlier grades regarding lines, angles and triangles, especially the similarity and congruence of triangles. 2. Investigate line segments joining the mid-points of two sides of a triangle. 3. Define the following special quadrilaterals: the kite, parallelogram, rectangle, rhombus, square and trapezium. Investigate and make conjectures about the properties of the sides, angles, diagonals and areas of these quadrilaterals. Prove these conjectures.	**Comments:** • Triangles are similar if their corresponding angles are equal, or if the ratios of their sides are equal: Triangles ABC and DEF are similar if $\hat{A}=\hat{D}$, $\hat{B}=\hat{E}$ and $\hat{C}=\hat{F}$. They are also similar if $\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}$. • We could define a parallelogram as a quadrilateral with two pairs of opposite sides parallel. Then we investigate and prove that the opposite sides of the parallelogram are equal, opposite angles of a parallelogram are equal, and diagonals of a parallelogram bisect each other. • It must be explained that a single counter example can disprove a Conjecture, but numerous specific examples supporting a conjecture do not constitute a general proof. **Example**: In quadrilateral KITE, KI = KE and IT = ET. The diagonals intersect at M. Prove that: 1. IM = ME and 2. KT is perpendicular to IE. As it is not obvious, first prove that $\Delta KIT\equiv\Delta KET$.

GRADE 10: TERM 3			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Analytical Geometry**	Represent geometric figures on a Cartesian co-ordinate system. Derive and apply for any two points $(x_1; y_1)$ and $(x_2; y_2)$ for the formulae for calculating the: 1. distance between the two points; 2. gradient of the line segment connecting the two points (and from that identify parallel and perpendicular lines); and 3. coordinates of the mid-point of the line segment joining the two points.	**Example**: Consider the points $P(2;5)$ and $Q(-3;1)$ in the Cartesian plane. 1.1 Calculate the distance PQ. 1.2 Find the coordinates of R if $M(-1;0)$ is the mid-point of PR. 1.3 Determine the coordinates of S if $PQRS$ is a parallelogram. 1.4 Is $PQRS$ a rectangle? Why or why not?
2	**Finance and growth**	Use the simple and compound growth formulae [$A = P(1+in)$ and $A = P(1+i)^n$] to solve problems, including interest, hire purchase, inflation, population growth and other real-life problems. Understand the implication of fluctuating foreign exchange rates (e.g. on the petrol price, imports, exports, overseas travel).	Note : Depreciation should also be taught : $A = P(1-in)$ and $A = P(1-i)^n$
2.5	**Statistics**	1. Revise measures of central tendency in ungrouped data. 2. Measures of central tendency in grouped data: calculation of mean estimate of grouped and ungrouped data and identification of modal interval and interval in which the median lies.	**Comment:** In grade 10, the intervals of grouped data should be given using inequalities, that is, in the form $0 \le x < 20$ rather than in the form $0-19,\ 20-29,\ \ldots$

<table>
<tr><td></td><td></td><td>3. Revision of range as a measure of dispersion and extension to include percentiles, quartiles, interquartile and semi interquartile range.
4. Five number summary (maximum, minimum and quartiles) and box and whisker diagram.
5. Use the statistical summaries (measures of central tendency and dispersion), and graphs to analyse and make meaningful comments on the context associated with the given data.</td><td>Example:
The mathematics marks of 200 grade 10 learners at a school can be summarised as follows:

<table><tr><th>Percentage obtained</th><th>Number of candidates</th></tr><tr><td>$0 \le x < 20$</td><td>4</td></tr><tr><td>$20 \le x < 30$</td><td>10</td></tr><tr><td>$30 \le x < 40$</td><td>37</td></tr><tr><td>$40 \le x < 50$</td><td>43</td></tr><tr><td>$50 \le x < 60$</td><td>36</td></tr><tr><td>$60 \le x < 70$</td><td>26</td></tr><tr><td>$70 \le x < 80$</td><td>24</td></tr><tr><td>$80 \le x < 100$</td><td>20</td></tr></table>
1. Calculate the approximate mean mark for the examination.
2. Identify the interval in which each of the following data items lie:
2.1 the median
2.2 the lower quartile
2.3 the upper quartile
2.4 the thirtieth percentile</td></tr>
<tr><td>1</td><td>Euclidean Geometry</td><td>Solve problems and prove riders using the properties of parallel lines, triangles and quadrilaterals.</td><td>Comment:
Use congruency and properties of quads, esp. parallelograms.
Formal proofs need to be used.

Example:
$EFGH$ is a parallelogram. Prove that $MFNH$ is a parallelogram.
</td></tr>
</table>

2	**Trigonometry**	Problems in two dimensions.	**Example:** Two flagpoles are 30 m apart. The one has height 10 m, while the other has height 15 m. Two tight ropes connect the top of each pole to the foot of the other. At what height above the ground do the two ropes intersect? What if the poles were a different distance apart?
1	**Measurement**	1. Revise the volume and surface areas of right-prisms and cylinders. 2. Study the effect on volume and surface area when multiplying any dimension by a constant factor *k*. 3. Calculate the volume and surface areas of spheres, right pyramids and right cones.	**Example:** The height of a cylinder is 10 cm, and the radius of the circular base is 2 cm. A hemisphere is attached to one end of the cylinder and a cone of height 2 cm to the other end. Calculate the volume and surface area of the solid, correct to the nearest cm^3 and cm^2 respectively. In case of pyramids, bases must either be an equilateral triangle or a square. Problem types must include composite figure.
2	**Probability**	1. The use of probability models to compare the relative frequency of events with the theoretical probability. 2. The use of Venn diagrams to solve probability problems, deriving and applying the following for any two events *A* and *B* in a sample space S: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$; A and B are mutually exclusive if $P(A \text{ and } B) = 0$; A and B are complementary if they are mutually exclusive ; and $P(A) + P(B) = 1$. Then $P(B) = P(\text{not}(A)) = 1 - P(A)$.	**Comment:** It generally takes a very large number of trials before the relative frequency of a coin falling heads up when tossed approaches 0,5. **Example:** A study was done to test how effective three different drugs, A, B and C were in relieving headaches. Over the period covered by the study, 80 patients were given the opportunity to use all two drugs. The following results were obtained: 40 reported relief from drug A 35 reported relief from drug B 40 reported relief from drug C 15 reported relief from both drugs A and B 21 reported relief from both drugs A and C 18 reported relief from drugs B and C 68 reported relief from at least one of the drugs 7 reported relief from all three drugs 1. Record this information in a Venn diagram. 2. How many subjects got no relief from any of the drugs? 3. How many subjects got relief from drugs A and B, but not C? 4. What is the probability that a randomly chosen subject got relief from at least one of the drugs?

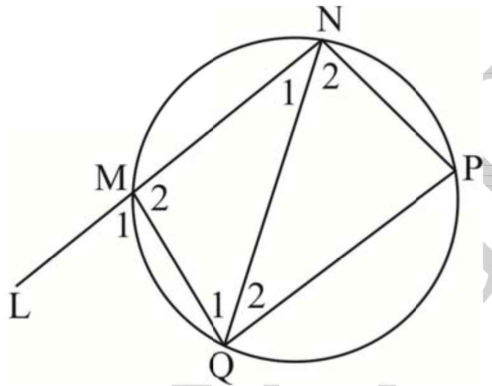
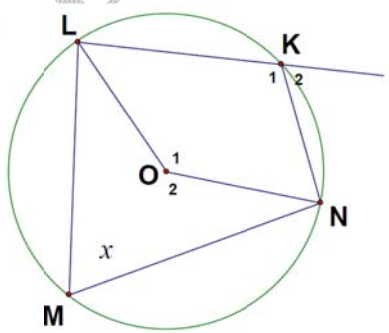
GRADE 11: TERM 1			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
3	**Exponents and surds**	1. Simplify expressions using the laws of exponents for rational exponents where $x^{\frac{p}{q}} = \sqrt[q]{x^p}$; $x > 0$; $q > 0$ and including Grade 10 content. 2. Add, subtract, multiply and divide simple surds.	**Examples:** 1. Determine the value of $9^{\frac{3}{2}}$, without the use of a calculator. 2. Simplify: $(3+\sqrt{2})(3-\sqrt{2})$.
3	**Equations and Inequalities**	1. Solve exponential equations and surd equations of the form $\sqrt{x+b} = ax+c,\ a,b,c \in \mathbb{Z}$ 2. Quadratic equations (by factorisation, by completion of the square and by using the quadratic formula). 3. Quadratic inequalities in one unknown (Interpret solutions graphically on number line, and interval notation). NB: It is recommended that the solving of equations in two unknowns is important to be used in other equations like hyperbola-straight line as this is normal in the case of graphs. 4. Equations in two unknown, one of which is linear and the other quadratic. 5. Nature of roots.	**Examples:** 1. 1.1 $2^{x+1} = \frac{1}{32}$ 1.2 $x^{\frac{2}{3}} = 4$ 1.3 $\sqrt{x+5} = 3x+1$ 2. 2.1 $x^2 + 2x = 5$ 2.2 $\frac{4}{x^2+4x+3} - \frac{4}{x-2} = \frac{3x+6}{x^2-x-2}$ 3. 3.1 Solve for x : $x^2 \le 4$ 3.2 Solve for x : $(x+1)(2x-3) \le 3$ 4. Given $(2x^2+3x-2)(x^2-3) = 0$ Solve for x when: 4.1 $x \in \mathbb{Z}$ 4.2 $x \in \mathbb{Q}$ 4.3 $x \in \mathbb{R}$ 5. Nature of roots. Recognition of the types of roots (see example 4).

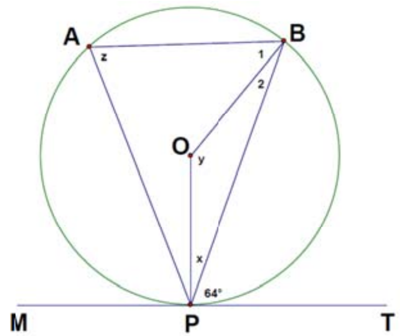
2	**Number patterns**	Patterns: Investigate number patterns including those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic.	**Examples:** 1. Write down the general term of the sequence: $\frac{1}{2}; \frac{4}{5}; \frac{9}{10}; \frac{16}{17}$ 2. Given the quadratic sequence 4 ; 9 ; 17 ; 28 ; 42 find the general term.
2.5	**Analytical Geometry**	Derive and apply: 1. the equation of a line though two given points; 2. the equation of a line through one point and parallel or perpendicular to a given line; and 3. the inclination (θ) of a line, where $m = \tan\theta$ is the gradient of the line $(0° \leq \theta < 180°)$.	**Example:** Given the points $A(2;5)$; $B(-3;-4)$ and $C(4;-2)$, determine: 1. the equation of the line AB; and 2. the size of $B\hat{A}C$.

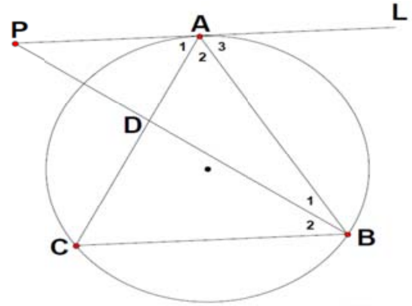
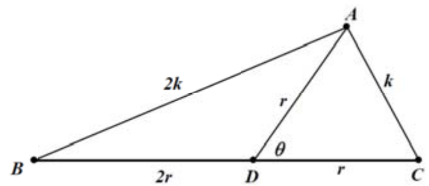
GRADE 11: TERM 2			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
4	**Functions**	1. Revise the effect of the parameters a and q and investigate the effect of p on the graphs of the functions defined by: 1.1 $y = f(x) = a(x+p)^2 + q$ 1.2 $y = f(x) = \frac{a}{x+p} + q$ 1.3 $y = f(x) = ab^{x+p} + q$ where $b > 0$, $b \neq 1$ 2. Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of a curve at a point. 3. Investigate the effect of the parameter k on the graphs of the functions defined by $y = \sin(kx)$, $y = \cos(kx)$ and $y = \tan(kx)$. 4. Investigate the effect of the parameter p on the graphs of the functions defined by $y = \sin(x+p)$, $y = \cos(x+p)$ and $y = \tan(x+p)$. 5. Draw sketch graphs defined by: $y = a\sin k(x+p)$, $y = a\cos k(x+p)$ and $y = \tan k(x+p)$ at most two parameters at a time.	**Comments:** • Once the effects of the parameters have been established, various problems need to be set: drawing sketch graphs, determining the defining equations of functions from sufficient data, making deductions from graphs. Real life applications of the prescribed functions should be studied. • Two parameters (maximum) at a time can be varied in tests or examinations. **Example**: (To be assessed in Paper 2 only.) Sketch the graphs defined by $y = -\frac{1}{2}\sin(x+30°)$ and $f(x) = \cos(2x-120°)$ on the same set of axes, where $-360° \leq x \leq 360°$.

4	**Trigonometry**	1. Derive and use the identities $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\theta \neq k.90°$, k an odd integer and $\sin^2\theta + \cos^2\theta = 1$. 2. Derive and use reduction formulae to simplify the following expressions: 2.1 $\sin(90° \pm \theta)$; $\cos(90° \pm \theta)$; 2.2 $\sin(180° \pm \theta)$; $\cos(180° \pm \theta)$; $\tan(180° \pm \theta)$; 2.3 $\sin(360° \pm \theta)$; $\cos(360° \pm \theta)$; $\tan(360° \pm \theta)$; and 2.4 $\sin(-\theta)$; $\cos(-\theta)$; $\tan(-\theta)$ 3. Determine for which values of a variable an identity holds. 4. Determine the general solutions of trigonometric equations. Also, determine solutions in specific intervals.	**Comment:** Teachers should explain where reduction formulae come from. **Examples:** 1. Prove that $\dfrac{1}{\tan\theta} + \tan\theta = \dfrac{\tan\theta}{\sin^2\theta}$. 2. For which values of θ is $\dfrac{1}{\tan\theta} + \tan\theta = \dfrac{\tan\theta}{\sin^2\theta}$ undefined. 3. Simplify $\dfrac{\cos(180° - x)\sin(x - 90°) - 1}{\tan^2(540° + x)\sin(90° x)\cos(-x)}$. 4. Determine the general solution of $\cos^2\theta + 3\sin\theta = -3$.

GRADE 11: TERM 3			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
1	**Measurement**	1. Revise the Grade 10 work.	Formulae for right prisms and cylinders will not be given in examinations.
3	**Euclidean Geometry**	Accept results established in earlier grades as axioms and also that a tangent to a circle is perpendicular to the radius, drawn to the point of contact. Then investigate and prove the theorems of the geometry of circles: • The line drawn from the centre of a circle perpendicular to a chord bisects the chord; • The perpendicular bisector of a chord passes through the centre of the circle; • The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre); • Angles subtended by a chord of the circle, on the same side of the chord, are equal; • The opposite angles of a cyclic quadrilateral are supplementary; • Two tangents drawn to a circle from the same point outside the circle are equal in length;	**Comments:** Proofs of the following theorems (acute angle case only) are examinable, their converses (where they exist) are not: Chords in circles - Line through centre and midpoint Angles in circles - Angle at centre = 2 x angle at circumference Cyclic Quadrilaterals - Opposite angles of cyclic quad Tangents to circles - Tan-chord theorem Use as results: • Angle subtended by a diameter is 90°. • Exterior angle of a cyclic quadrilateral is equal to int. opposite angle. • Angles in same segment are equal. • Two tangents drawn from same point outside a circle are equal. • The radius is perpendicular to the tangent at the point of contact. **Also:** • Diagrams for proofs will always be given. • Riders will place emphasis on proof, e.g. prove $x = 20°$; prove that $\hat{D} = 2x + y$; prove $AB \parallel CD$; prove $ABCD$ is a cyclic quad; name 4 other angles equal to x. • NO concurrency and NO proof by contradiction.

		• The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. Use the above theorems and their converses, where they exist, to solve riders.	**Examples:** 1.  Given: $\hat{N}_1 = 35°$ $\hat{N}_2 = 45°$ $\hat{Q}_1 = 50°$ (a) Write down, with a reason, the size of $\hat{M}_1$ (b) Prove $MN = NP$ 2. O is the centre of the circle above and $\hat{O}_1 = 2x$.  2.1 Determine $\hat{O}_2$ and $\hat{M}$ in terms of x. 2.2 Determine $\hat{K}_1$ and $\hat{K}_2$ in terms of x. 2.3 Determine $\hat{K}_1 + \hat{M}_2$. What do you notice? 2.4 Write down your observation regarding the measures of $\hat{K}_2$ and $\hat{M}$.

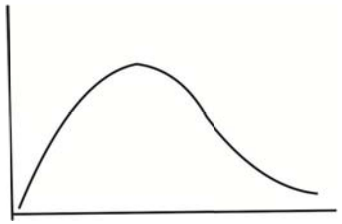
			3. O is the centre of the circle above and MPT is a tangent. Also, $OP \perp MT$. Determine, with reasons, x, y and z. 

			4. Given: $AB = AC$, $AP \parallel BC$ and $\hat{A}_2 = \hat{B}_2$.  Prove that: 4.1 PAL is a tangent to circle ABC; 4.2 AB is a tangent to circle ADP.
2	**Trigonometry**	1. Prove and apply the sine, cosine and area rules. 2. Solve problem in two dimensions using the sine, cosine and area rules.	**Comment:** The proofs of the sine, cosine and area rules are examinable. The proofs will be assessed in acute angle triangles only. The area rule may not assume the sine rule and vice versa. **Example:** In ΔABC, D is on BC, $A\hat{D}C = \theta$, $DA = DC = r$, $BD = 2r$, $AC = k$ and $BA = 2k$.  Show that $\cos\theta = \frac{1}{4}$.

2	**Finance, growth and decay**	1. Use simple and compound decay formulae: $A = P(1 - in)$ and $A = P(1 - i)^n$ to solve problems (including straight line depreciation and depreciation on a reducing balance). 2. The effect of different periods of compound growth and decay, including nominal and effective interest rates.	**Examples:** 1. The value of a piece of equipment depreciates from R10 000 to R5 000 in four years. What is the rate of depreciation if calculated on the: 1.1 straight line method; and 1.2 reducing balance? 2. Which is the better investment over a year or longer: 10,5% p.a. compounded daily or 10,55% p.a. compounded monthly? **Comment:** The use of a timeline to solve problems is a useful technique. 3. R50 000 is invested in an account which offers 8% p.a. interest compounded quarterly for the first 18 months. The interest then changes to 6% p.a. compounded monthly. Two years after the money is invested, R10 000 is withdrawn. How much will be in the account after 4 years? **Comment:** Stress the importance of not working with rounded answers, but of using the maximum accuracy afforded by the calculator right to the final answer when rounding might be appropriate.

GRADE 11: TERM 4			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
3	**Statistics**	1. Histograms 2. Frequency polygons 3. Ogives (cumulative frequency curves) 4. Variance and standard deviation of ungrouped data 5. Symmetric and skewed data 6. Identification of outliers	**Comments:** • Variance and standard deviation may be calculated using calculators. • Problems should cover topics related to health, social, economic, cultural, political and environmental issues. Symmetry and skewness should be done in context of a box and whisker diagram and histogram by observation, i.e. if values on one side tend to extend and 'tail off'. Also by comparing values of mean and median. **Examples:** 1. Consider the following statistics summary: | n | Mean | Median | σ | Minimum | Maximum | Q_1 | Q_3 | | 48 | 68,35 | 69,90 | 10,20 | 43,20 | 87,40 | 59,15 | 74,75 | (a) Draw the box and whisker plot for the data summarised in the table. (b) Would you describe the distribution of this data as skewed or symmetric? If skewed, in what direction? Explain your answer. (c) If an outlier is a value of greater than $Q_3 + 1,5 * IQR$ or less than $Q_1 - 1,5 * IQR$, where IQR is the interquartile range, show that there are no outliers in this data set. NB: Learners are expected to comment to the relationship between median and mean when discussing skewness.

2.

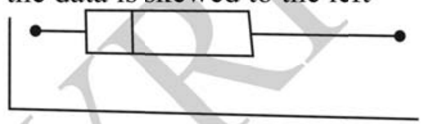


Choose the correct statement:

(a) the data is positively skewed

(b) the mean < median
the data is skewed to the left

(c)

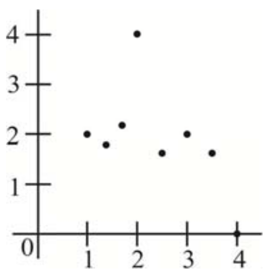


Comment :

- Outliers detection is important for effective modelling. Outliers should be excluded from such model fitting.
- Identification of outliers should be done in the context of a scatter plot as well as the box and whisker diagrams. Learners are not expected to memorise formulas for determining outliers.

Examples:

1. Consider the scatter plot drawn and answer the questions that follow.



			(a) Write down the co-ordinates of two points that are outliers. (b) Draw in a line of best fit. 4. **Example:** An outlier is any value that lies more than one and a half times the length of the box from either end of the box. That is, a data value is an outlier if it is less than $Q_1 - 1,5 \times IQR$ or greater than $Q_3 + 1,5 \times IQR$. Where Q_1 is the lower quartile, Q_3 is the upper quartile and IQR is the interquartile range. Find the outliers, if any for the following data set: 10 14 14 15 15 15 16 18
2	**Probability**	1. Revise and use tree diagrams and Venn diagrams to solve probability problems. 2. The use of tree diagrams for the probability of consecutive or simultaneous events which are not necessarily independent. 3. Revise the addition rule for mutually exclusive events: *P*(*A* or *B*) = *P*(*A*) + *P*(*B*), the complementary rule: *P*(not *A*) = 1 – *P*(*A*) and the identity *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*). 4. Dependent and independent events and the product rule for independent events: *P*(*A* and *B*) = *P*(*A*) × *P*(*B*)	**Examples:** 1. $P(A) = 0,45, P(B) = 0,3$ and $P(A \text{ or } B) = 0,615$. Are the events *A* and *B* mutually exclusive, independent or neither mutually exclusive nor independent? 2. What is the probability of throwing at least one six in four rolls of a regular six sided die? **Comment:** Venn Diagrams or Contingency tables can be used to study dependent and independent events. **Example:** In a group of 50 learners, 35 take Mathematics and 30 take History, while 12 take neither of the two. If a learner is chosen at random from this group, what is the probability that he/she takes both Mathematics and History?

GRADE 12: TERM 1			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
3	**Patterns, sequences, series**	1. Number patterns, including arithmetic and geometric sequences and series 2. Sigma notation 3. Derivation and application of the formulae for the sum of arithmetic and geometric series: 3.1 $S_n = \frac{n}{2}[2a+(n-1)d]$; $S_n = \frac{n}{2}(a+l)$ 3.2 $S_n = \frac{a(r^n-1)}{r-1}$; $(r \neq 1)$; and 3.3 $S_\infty = \frac{a}{1-r}$; $(-1 < r < 1)(r \neq 1)$	**Comment:** Derivation of the formulae is examinable. **Examples:** 1. 1.1 Write down the first terms of the sequence with general term $T_k = \frac{1}{3k-1}$ 1.2 $\sum_{k=0}^{3}(3k-1)$ 2. Determine the 5th term of the geometric sequence of which the 8th term is 6 and the 12th term is 14. 3. Determine the largest value of n such that $\sum_{i=1}^{n}(3i-2) < 2000$ 4. Show that $0,\dot{9} = 1$
3	**Functions**	1. Definition of a *function*. 2. General concept of the *inverse of a function* and how the domain of the function may need to be restricted (in order to obtain a one-to-one function) to ensure that the inverse is a function. 3. Determine the sketch graphs of the inverses of the functions defined by $y = ax + q$; $y = ax^2$ $y = b^x$; $(b > 0, b \neq 1)$	**Examples:** 1. Consider the function f where $f(x) = 3x - 1$. 1.1 Write down the domain and range of f. 1.2 Show that f is a one-to-one relation. 1.3 Determine the inverse function f^{-1}. 1.4 Sketch the graphs of the functions f, f^{-1} and $y = x$ line on the same set of axes. 2. Repeat Question 1 for the function $f(x) = -x^2$ and $x \leq 0$.

		Focus on the following characteristics: domain and range, intercepts with the axes, turning points, minima, maxima, asymptotes (horizontal and vertical), shape and symmetry, average gradient (average rate of change), intervals on which the function increases/decreases.	**Comments:** 1. Do not confuse the inverse function $f^{-1}(x)$ with the reciprocal $\frac{1}{f(x)}$. For example, for the function where $f(x)=\sqrt{x}$, the reciprocal is $\frac{1}{\sqrt{x}}$, while $f^{-1}(x)=x^2$ for $x\geq 0$. 2. Note that the notation $f^{-1}(x)=\ldots$ is used only for one-to-one relation and must not be used for inverses of many-to-one relations, since in these cases the inverses are not functions.
1	**Functions: exponential and logarithmic**	1. Revision of the exponential function and the exponential laws and graph of the function defined by $y=b^x$ where $b>0$ and $b\neq 1$. 2. Understand the definition of a logarithm: $y=\log_b x \Leftrightarrow x=b^y$, where $b>0$ and $b\neq 1$. 3. The graph of the function define $y=\log_b x$ for both the cases $0<b<1$ and $b>1$.	**Comments:** 1. Make sure learners know the difference between the two functions defined by $y=b^x$ and $y=x^b$ where b is a positive (constant) real number. 2. Manipulation involving the logarithmic laws will not be examined. 3. Context involving logarithms related to finance, growth and decay can be examined. **Examples:** 1. Solve for x: 75 $(1{,}025)^{x-1}=300$ 2. Let $f(x)=a^x$, $a>0$. 2.1 Determine a if the graph of f goes through the point $(2;\frac{25}{16})$. 2.2 Determine the function f^{-1}. 2.3 For which values of x is $f^{-1}(x)>-1$? 2.4 Determine the function h if the graph of h is the reflection of the graph of f through the y-axis. 2.5 Determine the function k if the graph of k is the reflection of the graph of f through the x-axis. 2.6 Determine the function p if the graph of p is obtained by shifting the graph of f two units to the left. 2.7 Write down the domain and range for each of the functions f, f^{-1}, h, k and p. 2.8 Represent all these functions graphically.

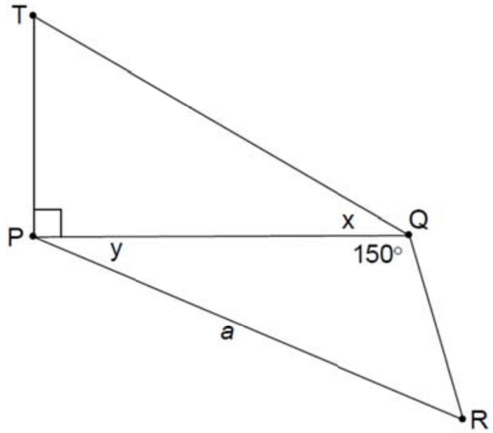
<table>
<tr>
<td>2</td>
<td>Finance, growth and decay</td>
<td>1. Solve problems involving present value and future value annuities.
2. Make use of logarithms to calculate the value of n, the time period, in the equations $A = P(1+i)^n$ or $A = P(1-i)^n$.
3. Critically analyse investment and loan options and make informed decisions as to best option(s) (including pyramid schemes).</td>
<td>Comment:
1. Derivation of the formulae for present and future values using the geometric series formula $Sn = \frac{a(r^n - 1)}{r-1}; r \neq 1$, will not be required for examination purposes, but should be part of the teaching process to ensure that the learners understand where the formulae come from.
The two annuity formulae: $F = \frac{x((1+i)^n - 1)}{i}$ and $P = \frac{x(1-(1+i)^{-n})}{i}$ hold only when payment commences one period from the present and ends after n periods.

2. NB. No variations of the above formulae will be examinable.

3. The use of a timeline to analyse problems is a useful technique.

Examples:
1. Given that a population increased from 120 000 to 214 000 in 10 years, at what annual (compound) rate did the population grow?
2. In order to buy a car, John takes out a loan of R25 000 from the bank. The bank charges an annual interest rate of 11% p.a. compounded monthly. The instalments start a month after he has received the money from the bank.
2.1 Calculate his monthly instalments if he has to pay back the loan over a period of 5 years.
2.2 Calculate the outstanding balance of his loan after two years (immediately after the 24th instalment).</td>
</tr>
<tr>
<td>2</td>
<td>Trigonometry</td>
<td>Compound angle identities:
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
$\sin 2\alpha = 2\sin\alpha\cos\alpha$
$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$</td>
<td>Comment: The derivation of the compound and double angle formulae will not be required for examination purposes, but should be part of the teaching process.

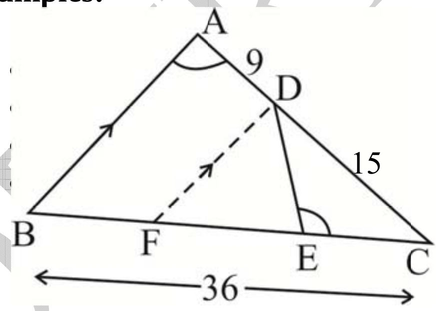
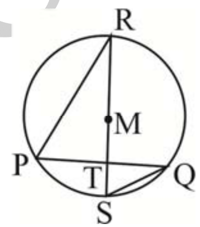
Examples:
1. Determine the general solution of $\sin 2x + \cos x = 0$.
2. Prove that $\frac{1+\sin 2x}{\cos 2x} = \frac{\cos x + \sin x}{\cos x - \sin x}$.
3. Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$</td>
</tr>
</table>

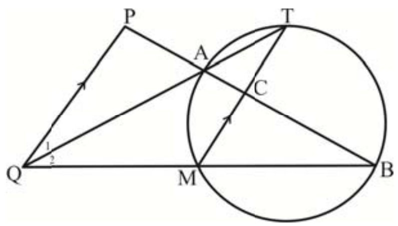
GRADE 12: TERM 2			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Trigonometry**	1. Solve problems in two and three dimensions.	**Examples:**  1. TP is a tower. Its foot, P, and the points Q and R are on the same horizontal plane. From Q the angle of elevation to the top of the building is x. Furthermore, $P\hat{Q}R = 150°$, $Q\hat{P}R = y$ and the distance between P and R is a metres. Prove that $TP = a\tan x(\cos y - \sqrt{3}\sin y)$ 2. In ΔABC, $AD \perp BC$. Prove that: a. $a = b\cos C + c\cos B$ where $a = BC$; $b = AC$ and $c = AB$. b. $\frac{\cos B}{\cos C} = \frac{c - b\cos A}{b - c\cos A}$ (on condition that $\hat{C} \neq 90°$). c. $\tan A = \frac{a\sin C}{b - a\cos C}$ (on condition that $\hat{A} \neq 90°$). d. $a + b + c = (b+c)\cos A + (c+a)\cos B + (a+b)\cos C$.
1	**Functions: Polynomials**	Factorise third-degree polynomials. Apply the Remainder and Factor Theorems to polynomials of degree at most 3 (no proofs required).	**Comment**: Any method may be used to factorise third degree polynomials in the examinations, but the teaching process should include examples which require the Factor Theorem. **Example:** Solve for x: $x^3 + 8x^2 + 17x + 10 = 0$

3	**Differential Calculus**	1. An intuitive understanding of the limit concept, in the context of approximating the rate of change or gradient of a function at a point. 2. Use limits to define the derivative of a function f at any x: $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ Generalise to find the derivative of f at any point x in the domain of f, i.e. define the derivative function $f'(x)$ of the function $f(x)$. Understand intuitively that $f'(a)$ is the gradient of the tangent to the graph of f at the point with x-coordinate a. 3. Using the definition (first principle), find the derivative, $f'(x)$ for a, b and c constants: (a) $f(x) = ax^2 + bx + c$; (b) $f(x) = ax^3$; (c) $f(x) = \frac{a}{x}$; (d) $f(x) = c$ 4. Use the formula $\frac{d}{dx}(ax^n) = anx^{n-1}$, (for any real number n) together with the rules:	**Comment:** Differentiation from first principles will be examined on any of the types described in 3 (a), (b) and (c) in the 'Curriculum statement'. **Examples:** 1. In each of the following cases, find the derivative of $f(x)$ at the point where $x = -1$, using the definition of the derivative: 1.1 $f(x) = x^2 + 2$ 1.2 $f(x) = \frac{1}{2}x^2 + x - 2$ 1.3 $f(x) = -x^3$ 1.4 $f(x) = -\frac{2}{x}$ **Caution**: Care should be taken not to apply the sum rule for differentiation (4(a)) in a similar way to products. a. Determine $\frac{d}{dx}((x+1)(x-1))$. b. Determine $\frac{d}{dx}(x+1) \times \frac{d}{dx}(x-1)$. c. Write down your observation. 2. Use differentiation rules to do the following: 2.1 Determine $f'(x)$ if $f(x) = (x+2)^2$ 2.2 Determine $f'(x)$ if $f(x) = \frac{(x+2)^3}{\sqrt{x}}$ 2.3 Determine $\frac{dy}{dt}$ if $y = \frac{t^2-1}{2t+2}$ 2.4 Determine $f'(\theta)$ if $f(\theta) = (\theta^{3/2} - 3\theta^{-1/2})^2$

		(a) $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$ and (b) $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)]$ (k a constant) 5. Find equations of tangents to graphs of functions. 6. Introduce the second derivative $f''(x) = \frac{d}{dx}(f'(x))$ of $f(x)$ and how it determines the concavity of a function. 7. Sketch graphs of cubic polynomial functions using differentiation to determine the co-ordinate of stationary points, and points of inflection (where concavity changes). Also, determine the x-intercepts of the graph using the factor theorem and other techniques. 8. Solve practical problems concerning optimisation and rate of change, including calculus of motion.	3. Determine the equation of the tangent to the graph defined by $y = (2x+1)^2(x+2)$ where $x = \frac{3}{4}$. 4. Sketch the graph defined by $y = -x^3 + 4x^2 - x$ by: 4.1 finding the intercepts with the axes; 4.2 finding maxima, minima and the co-ordinates of the point of inflection; (Remember: To understand points of inflection, an understanding of concavity is necessary. This is where the second derivative plays a role.) 5. The radius of the base of a circular cylindrical can is x cm, and its volume is 430 cm^3. 5.1 Determine the height of the can in terms of x. 5.2 Determine the area of the material needed to manufacture the can (that is, determine the total surface area of the can) in terms of x. 5.3 Determine the value of x for which the least amount of material is needed to manufacture such a can. 5.4 If the cost of the material is R500 per m^2, what is the cost of the cheapest can (labour excluded)? 6. The motion of a particle is described by $S(t) = st^2 - 3^t$. 6.1 Find an expression for the velocity $V(t) = S^1(t)$. 6.2 Given acceleration $a(t) = V^1(t)$, find the acceleration of the particle.
2	**Analytical Geometry**	1. The equation $(x-a)^2 + (y-b)^2 = r^2$ defines a circle with radius r and centre $(a;b)$. 2. Determination of the equation of a tangent to a given circle.	**Examples:** 1. Determine the equation of the circle with centre $(-1;2)$ and radius $\sqrt{6}$. 2. Determine the equation of the circle which has the line segment with endpoints $(5;3)$ and $(-3;6)$ as diameter. 3. Determine the equation of a circle with a radius of 6 units, which intersects the x-axis at $(-2;0)$ and the y-axis at $(0;3)$. How many such circles are there?

			4. Determine the equation of the tangent that touches the circle defined by $x^2 - 2x + y^2 + 4y = 5$ at the point $(-2;-1)$. 5. The line with the equation $y = x + 2$ intersects the circle defined by $x^2 + y^2 = 20$ at A and B. 5.1 Determine the co-ordinates of A and B. 5.2 Determine the length of chord AB. 5.3 Determine the co-ordinates of M, the midpoint of AB. 5.4 Show that $OM \perp AB$ where O is the origin. 5.5 Determine the equations of the tangents to the circle at the points A and B. 5.6 Determine the co-ordinates of the point C where the two tangents in (5.5) intersect. 5.7 Verify that $CA = CB$. 5.8 Determine the equations of the two tangents to the circle, both parallel to the line with the equation $y = -2x + 4$. 6. Determine the length of the tangent drawn from $A(-2;5)$ to the circle with equation $x^2 + (y-10)^2 = 4$. 7. Given the circles: $x^2 + y^2 = 1$ and $(x-3)^2 + (y-4)^2 = 16$ Show that the circles touch each other.

GRADE 12: TERM 3			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Euclidean Geometry**	1. Revise earlier work on the necessary and sufficient conditions for polygons to be similar. 2. Prove (accepting results established in earlier grades): • that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Mid-point Theorem as a special case of this theorem); • that equiangular triangles are similar; • that triangles with sides in proportion are similar; and • the Pythagorean Theorem by similar triangles	**Comments:** • Riders will concentrate on proof of the following types: (a)Prove $\Delta ABC \mathbin{\|\|\|} \Delta DEF$ (b)Prove AB.PR = AC.PQ (i.e. must know to rearrange and identify which triangles to prove similar) • Riders will also concentrate on numerical calculations. • Combinations of Grade 12 with Grade 11 Geometry will be kept at a routine level. (old 'standard Grade' level) **Examples:** 1.  1.1 Prove that $\Delta CDE \mathbin{\|\|\|} \Delta CBA$ 1.2. Calculate (a) EC (b) CF (c) FE 2.  M is the circle centre $RMS \perp PQ$ $PQ = 4x$; $TS = x$ and $RT = 150$ mm



3.

3.1 If $\hat{Q}_1 = x$ name two other angles equal to, giving reasons.

3.2 Prove that ΔPBQ|||ΔPQA

3.3 Deduce that $PQ = \sqrt{PA.PB}$

3.4 If PB=11,25cm; QM=9cam and MB = 6cm calculate the length of CB.

Comment :

The following proofs are examinable :

Proportion

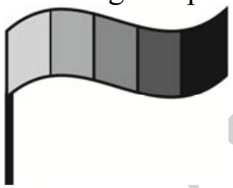
- Line parallel to one side of triangle

Similar Triangles

- Triangles equiangular
- Perpendicular from right angle to hypotenuse

<table>
<tr>
<td>1</td>
<td>Statistics (regression and correlation)</td>
<td>1. Revise symmetric and skewed data.
2. Use statistical summaries, scatterplots, regression (in particular the least squares regression line) and correlation to analyse and make meaningful comments on the context associated with given bivariate data, including interpolation, extrapolation and discussions on skewness.</td>
<td>Comments:
• Ability to suggest a function type of best fit by inspection, but find least squares regression line $y = a + bx$ using technology (calculator).
• Know that $(\overline{x};\overline{y})$ lies on line of best fit.
• Identify the correlation coefficient (r) as the value that quantifies the strength and direction of the linear relationship between the variables in a set of bivariate data. Interpretation of r values between $-1 \leq r \leq 1$
• Beware: Correlation does not imply causation.

Examples:
1. The following table summarises the number of revolutions x (per minute) and the corresponding power output y (horse power) of a Diesel engine:
<table>
<tr><td>x</td><td>400</td><td>500</td><td>600</td><td>700</td><td>750</td></tr>
<tr><td>y</td><td>580</td><td>1030</td><td>1420</td><td>1880</td><td>2100</td></tr>
</table>
1.1 Find the least squares regression line $y = a + bx$
1.2 Use this line to estimate the power output when the engine runs at 800 m.
1.3 Roughly how fast is the engine running when it has an output of 1200 horse power?
2. An r value for a certain set of bivariate is calculated to have a value equal to –0,243. (There may be more than one.) Select the correct interpretation(s) of the value r.
2.1 a strong positive relationship
2.2 a weak relationship
2.3 a moderate negative relationship
2.4 an indication that as one variable increases the other decreases
2.5 a strong negative relationship
2.6 no relationship</td>
</tr>
</table>

2	**Counting and probability**	1. Revise: • dependent and independent events; • the product rule for independent events: P(A and B) = P(A) × P(B). • the sum rule for mutually exclusive events A and B: $P(A \text{ or } B) = P(A) + P(B)$ • the identity: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ • the complementary rule: $P(\text{not } A) = 1 - P(A)$ 2. Probability problems using Venn diagrams, trees, two-way contingency tables and other techniques (like the fundamental counting principle) to solve probability problems (where events are not necessarily independent). 3. Apply the fundamental counting principle to solve probability problems.	**Examples:** 1. Given $P(A) = 0{,}4$; $P(B) = 0{,}25$ and $P(A \cap B) = 0{,}1$ 1.1 Determine if A and B are independent events. 1.2 Evaluate $P(A \cup B)$ 2. How many three-character codes can be formed if the first character must be a letter and the second two characters must be different digits? 2.1 if repetition is allowed 2.2 if repetition is not allowed 3. A flag comprises 5 vertical bands  Bands are available in 7 colours. Determine the number of different flags that can be created if: 3.1 Each band is a different colour. 3.2 The first, third and fifth bands are the same colour. 4. What is the probability that a random arrangement of the letters BAFANA starts and ends with an 'A'? 5. Four different glasses and 5 different bottles are arranged on a shelf. How many arrangements are there if they are placed 5.1 at random? 5.2 all the glasses together and all bottles together? in alternating positions? 6. Four red discs, 2 blue discs and 5 yellow discs are places in a bag. 2 discs are randomly chosen without replacement. Find the probability that: 6.1 both are red 6.2 both are the same colour 6.3 both are not blue 6.4 you get one red and one blue disc **Comment:** Questions needing permutations or combinations are not examinable.